Hoofdstuk 6: Goniometrische formules.

6.1 Eenheidscirkel en radiaal

Opgave 1:

a. $\sin 65° = \dfrac{PQ}{1}$

$PQ = \sin 65° = 0{,}91$

$OQ = \cos 65° = 0{,}42$

b. $P(0{,}42;0{,}91)$

c. $\angle POQ = 65°$

$PQ = 0{,}91$

$OQ = 0{,}42$

$P(-0{,}42;0{,}91)$

d. $\cos 115° = -0{,}42$

$\sin 115° = 0{,}91$

$\cos 115° = -\cos 65°$

$\sin 115° = \sin 65°$

Opgave 2:

a. 0
b. 1
c. 1
d. 0
e. -1
f. 0
g. 0
h. 1
i. 1
j. 0
k. 0
l. -1

Opgave 3:

a. $\sin 210° = -\sin 30° = -\frac{1}{2}$

b. $\cos 210° = -\cos 30° = -\frac{1}{2}\sqrt{3}$

c. $\sin(-135°) = -\sin 45° = -\frac{1}{2}\sqrt{2}$

d. $\cos(-135°) = -\cos 45° = -\frac{1}{2}\sqrt{2}$

e. $\sin 300° = -\sin 60° = -\frac{1}{2}\sqrt{3}$

f. $\cos 300° = \cos 60° = \frac{1}{2}$

Opgave 4:

a. klopt

b. $x_P = \cos 110° = -0{,}34$ $\quad y_P = \sin 110° = 0{,}94$

$x_Q = \cos 200° = -0{,}94$ $\quad y_Q = \sin 200° = -0{,}34$

$x_R = \cos(-102°) = -0{,}21$ $\quad$ $y_R = \sin(-102°) = -0{,}98$

$x_S = \cos(-50°) = 0{,}64$ $\quad$ $y_S = \sin(-50°) = -0{,}77$

Opgave 5:

$x_B = 2\cos 72° = 0{,}62$ $\quad$ $y_B = 2\sin 72° = 1{,}90$

$x_C = 2\cos 144° = -1{,}62$ $\quad$ $y_C = 2\sin 144° = 1{,}18$

$x_D = 2\cos 216° = -1{,}62$ $\quad$ $y_D = 2\sin 216° = -1{,}18$

$x_E = 2\cos 288° = -0{,}62$ $\quad$ $y_E = 2\sin 288° = -1{,}90$

Opgave 6:

a. $omtrek = 2\pi r = 2 \cdot \pi \cdot 1 = 2\pi$

b. bij $90°$ hoort een kwart cirkel dus $boog = \frac{1}{4} \cdot 2\pi = \frac{1}{2}\pi$

c. $\frac{1}{2} \cdot omtrek = \frac{1}{2} \cdot 2\pi = \pi$

d. $a \cdot omtrek = 1\frac{1}{2}\pi$

$a \cdot 2\pi = 1\frac{1}{2}\pi$

$a = \frac{3}{4}$

dus $\frac{3}{4} \cdot 360° = 270°$

Opgave 7:

a. $30°$

b. $45°$

c. $360°$

d. $\dfrac{360}{\pi} = 114{,}6°$

e. $225°$

f. $\dfrac{225}{\pi} = 71{,}6°$

g. $-400°$

h. $-127{,}3°$

Opgave 8:

a. 2π

b. $\frac{1}{6}\pi$

c. $\frac{1}{4}\pi$

d. $\frac{1}{3}\pi$

e. $\frac{1}{2}\pi$

f. $\frac{3}{4}\pi$

g. $1\frac{2}{3}\pi$

h. $1\frac{1}{6}\pi$

Opgave 9:

a. $\dfrac{10}{360} \cdot 2\pi = 0{,}17$

b. $\frac{57,3}{360} \cdot 2\pi = 1,00$

c. $\frac{1030}{360} \cdot 2\pi = 17,98$

d. $\frac{90}{360} \cdot 2\pi = 1,57$

Opgave 10:

a. $-0,38$

b. $0,81$

c. $0,59$

d. $0,72$

e. $0,31$

f. $0,25$

Opgave 11:

a. $x_P = \cos 5 = 0,28$
$y_P = \sin 5 = -0,96$

b. $x_P = \cos 6 = 0,96$
$y_P = \sin 6 = -0,28$

c. $x_P = \cos 20 = 0,41$
$y_P = \sin 20 = 0,91$

Opgave 12:

a. $\frac{1}{2}\sqrt{3}$

b. $\frac{1}{2}\sqrt{2}$

Opgave 13:

a. $\sin \frac{3}{4}\pi = \sin \frac{1}{4}\pi = \frac{1}{2}\sqrt{2}$

b. $\cos \frac{7}{6}\pi = -\cos \frac{1}{6}\pi = -\frac{1}{2}\sqrt{3}$

c. $\sin 1\frac{1}{3}\pi = -\sin \frac{1}{3}\pi = -\frac{1}{2}\sqrt{3}$

d. $\cos \frac{5}{3}\pi = \cos \frac{1}{3}\pi = \frac{1}{2}$

e. $\cos 1\frac{1}{3}\pi = -\cos \frac{1}{3}\pi = -\frac{1}{2}$

f. $\sin -\frac{1}{4}\pi = -\sin \frac{1}{4}\pi = -\frac{1}{2}\sqrt{2}$

Opgave 14:

a. $\alpha = \frac{1}{3}\pi \quad \vee \quad \alpha = \frac{2}{3}\pi$

b. $\alpha = \frac{2}{3}\pi \quad \vee \quad \alpha = 1\frac{1}{3}\pi$

c. $\alpha = 1\frac{1}{4}\pi \quad \vee \quad \alpha = 1\frac{3}{4}\pi$

d. $\alpha = \frac{1}{2}\pi \quad \vee \quad \alpha = 1\frac{1}{2}\pi$

e. $\alpha = \frac{1}{6}\pi \quad \vee \quad \alpha = 1\frac{5}{6}\pi$

f. $\alpha = \frac{1}{4}\pi \quad \vee \quad \alpha = 1\frac{1}{4}\pi$

6.2 Goniometrische vergelijkingen.

Opgave 15:

Bij $(0,1)$: $\ldots, -3\frac{1}{2}\pi, -1\frac{1}{2}\pi, \frac{1}{2}\pi, 2\frac{1}{2}\pi, 4\frac{1}{2}\pi, \ldots$

Bij $(0,-1)$: $\ldots, -4\frac{1}{2}\pi, -2\frac{1}{2}\pi, -\frac{1}{2}\pi, 1\frac{1}{2}\pi, 3\frac{1}{2}\pi, \ldots$

Opgave 16:

a. $\sin(3x - \frac{1}{2}\pi) = 0$

$3x - \frac{1}{2}\pi = 0 + k \cdot \pi$

$3x = \frac{1}{2}\pi + k \cdot \pi$

$x = \frac{1}{6}\pi + k \cdot \frac{1}{3}\pi$

b. $\cos(\frac{1}{2}x - \frac{1}{6}\pi) = 0$

$\frac{1}{2}x - \frac{1}{6}\pi = \frac{1}{2}\pi + k \cdot \pi$

$\frac{1}{2}x = \frac{2}{3}\pi + k \cdot \pi$

$x = 1\frac{1}{3}\pi + k \cdot 2\pi$

c. $\sin^2 x - \sin x = 0$

$\sin x(\sin x - 1) = 0$

$\sin x = 0 \quad \vee \quad \sin x = 1$

$x = 0 + k \cdot \pi \quad \vee \quad x = \frac{1}{2}\pi + k \cdot 2\pi$

d. $\cos^2 2x + \cos 2x = 0$

$\cos 2x(\cos 2x + 1) = 0$

$\cos 2x = 0 \quad \vee \quad \cos 2x = -1$

$2x = \frac{1}{2}\pi + k \cdot \pi \quad \vee \quad 2x = \pi + k \cdot 2\pi$

$x = \frac{1}{4}\pi + k \cdot \frac{1}{2}\pi \quad \vee \quad x = \frac{1}{2}\pi + k \cdot \pi$

Opgave 17:

a. $\sin 2x = 1$

$2x = \frac{1}{2}\pi + k \cdot 2\pi$

$x = \frac{1}{4}\pi + k \cdot \pi$

$\sin 2x = -1$

$2x = -\frac{1}{2}\pi + k \cdot 2\pi$

$x = -\frac{1}{4}\pi + k \cdot \pi$

b. $\ldots, -1\frac{1}{4}\pi, -\frac{1}{4}\pi, \frac{3}{4}\pi, 1\frac{3}{4}\pi, \ldots$

c. als je de twee rijtjes oplossingen samenvoegt krijg je:

$\ldots, -1\frac{3}{4}\pi, -1\frac{1}{4}\pi, -\frac{3}{4}\pi, -\frac{1}{4}\pi, \frac{1}{4}\pi, \frac{3}{4}\pi, 1\frac{1}{4}\pi, 1\frac{3}{4}\pi, \ldots$

Dat is: $x = \frac{1}{4}\pi + k \cdot \frac{1}{2}\pi$

Opgave 18:

a. $\cos^2(x - \frac{1}{5}\pi) = 1$

$\cos(x - \frac{1}{5}\pi) = 1 \quad \vee \quad \cos(x - \frac{1}{5}\pi) = -1$

$x - \frac{1}{5}\pi = 0 + k \cdot 2\pi \quad \vee \quad x - \frac{1}{5}\pi = \pi + k \cdot 2\pi$

$x = \frac{1}{5}\pi + k \cdot 2\pi \quad \vee \quad x = 1\frac{1}{5}\pi + k \cdot 2\pi$

samen $x = \frac{1}{5}\pi + k \cdot \pi$

b. $\sin^2(2x - \frac{1}{4}\pi) = 1$

$\sin(2x - \frac{1}{4}\pi) = 1 \quad \vee \quad \sin(2x - \frac{1}{4}\pi) = -1$

$2x - \frac{1}{4}\pi = \frac{1}{2}\pi + k \cdot 2\pi \quad \vee \quad 2x - \frac{1}{4}\pi = -\frac{1}{2}\pi + k \cdot 2\pi$

$2x = \frac{3}{4}\pi + k \cdot 2\pi \quad \vee \quad 2x = -\frac{1}{4}\pi + k \cdot 2\pi$

$x = \frac{3}{8}\pi + k \cdot \pi \quad \vee \quad x = -\frac{1}{8}\pi + k \cdot \pi$

samen $x = \frac{3}{8}\pi + k \cdot \frac{1}{2}\pi$

c. $\sin^3 x - \sin x = 0$

$\sin x(\sin^2 x - 1) = 0$

$\sin x = 0 \quad \vee \quad \sin^2 x = 1$

$\sin x = 0 \quad \vee \quad \sin x = 1 \quad \vee \quad \sin x = -1$

$x = 0 + k \cdot \pi \quad \vee \quad x = \frac{1}{2}\pi + k \cdot 2\pi \quad \vee \quad x = -\frac{1}{2}\pi + k \cdot 2\pi$

samen $x = 0 + k \cdot \frac{1}{2}\pi$

d. $\cos^3 2x - \cos 2x = 0$

$\cos 2x(\cos^2 2x - 1) = 0$

$\cos 2x = 0 \quad \vee \quad \cos^2 2x = 1$

$\cos 2x = 0 \quad \vee \quad \cos 2x = 1 \quad \vee \quad \cos 2x = -1$

$2x = \frac{1}{2}\pi + k \cdot \pi \quad \vee \quad 2x = 0 + k \cdot 2\pi \quad \vee \quad 2x = \pi + k \cdot 2\pi$

samen $2x = 0 + k \cdot \frac{1}{2}\pi$

dus $x = 0 + k \cdot \frac{1}{4}\pi$

Opgave 19:

a. $\sin(4x - \frac{1}{3}\pi) = 1$

$4x - \frac{1}{3}\pi = \frac{1}{2}\pi + k \cdot 2\pi$

$4x = \frac{5}{6}\pi + k \cdot 2\pi$

$x = \frac{5}{24}\pi + k \cdot \frac{1}{2}\pi$

b. $\cos 4\pi x = 1$

$4\pi x = 0 + k \cdot 2\pi$

$x = 0 + k \cdot \frac{1}{2}$

c. $\sin^2(\frac{1}{4}\pi x) = 1$

$\sin \frac{1}{4}\pi x = 1 \quad \vee \quad \sin \frac{1}{4}\pi x = -1$

$\frac{1}{4}\pi x = \frac{1}{2}\pi + k \cdot 2\pi \quad \vee \quad \frac{1}{4}\pi x = -\frac{1}{2}\pi + k \cdot 2\pi$

samen $\frac{1}{4}\pi x = \frac{1}{2}\pi + k \cdot \pi$

dus $x = 2 + k \cdot 4$

d. $\sin 2x \cdot \cos 2x + \sin 2x = 0$

$\sin 2x(\cos 2x + 1) = 0$

$\sin 2x = 0 \quad \vee \quad \cos 2x = -1$

$2x = 0 + k \cdot \pi \quad \vee \quad 2x = \pi + k \cdot 2\pi$

$x = 0 + k \cdot \frac{1}{2}\pi \quad \vee \quad x = \frac{1}{2}\pi + k \cdot \pi$

dus $x = 0 + k \cdot \frac{1}{2}\pi$

Opgave 20:

a. $\sin \frac{1}{6}\pi = \frac{1}{2}$

b. $\sin 2\frac{1}{6}\pi = \sin \frac{1}{6}\pi = \frac{1}{2}$

$\sin 4\frac{1}{6}\pi = \sin \frac{1}{6}\pi = \frac{1}{2}$

c. $\sin \frac{5}{6}\pi = \frac{1}{2}$

d. $\sin 2\frac{5}{6}\pi = \sin \frac{5}{6}\pi = \frac{1}{2}$

$\sin -1\frac{1}{6}\pi = \sin \frac{5}{6}\pi = \frac{1}{2}$

Opgave 21:

a. $2\sin \frac{1}{2}x = 1$

$\sin \frac{1}{2}x = \frac{1}{2}$

$\frac{1}{2}x = \frac{1}{6}\pi + k \cdot 2\pi \quad \vee \quad \frac{1}{2}x = \frac{5}{6}\pi + k \cdot 2\pi$

$x = \frac{1}{3}\pi + k \cdot 4\pi \quad \vee \quad x = \frac{5}{3}\pi + k \cdot 4\pi$

b. $2\cos(x - \frac{1}{3}\pi) = 1$

$\cos(x - \frac{1}{3}\pi) = \frac{1}{2}$

$x - \frac{1}{3}\pi = \frac{1}{3}\pi + k \cdot 2\pi \quad \vee \quad x - \frac{1}{3}\pi = -\frac{1}{3}\pi + k \cdot 2\pi$

$x = \frac{2}{3}\pi + k \cdot 2\pi \quad \vee \quad x = 0 + k \cdot 2\pi$

c. $2\sin(2x - \frac{1}{4}\pi) = -\sqrt{3}$

$\sin(2x - \frac{1}{4}\pi) = -\frac{1}{2}\sqrt{3}$

$2x - \frac{1}{4}\pi = -\frac{1}{3}\pi + k \cdot 2\pi \quad \vee \quad 2x - \frac{1}{4}\pi = 1\frac{1}{3}\pi + k \cdot 2\pi$

$2x = -\frac{1}{12}\pi + k \cdot 2\pi \quad \vee \quad 2x = 1\frac{7}{12}\pi + k \cdot 2\pi$

$x = -\frac{1}{24}\pi + k \cdot \pi \quad \vee \quad x = \frac{19}{24}\pi + k \cdot \pi$

d. $2\cos(3x - \pi) = -1$

$\cos(3x - \pi) = -\frac{1}{2}$

$3x - \pi = \frac{2}{3}\pi + k \cdot 2\pi \quad \vee \quad 3x - \pi = -\frac{2}{3}\pi + k \cdot 2\pi$

$3x = 1\frac{2}{3}\pi + k \cdot 2\pi \quad \vee \quad 3x = \frac{1}{3}\pi + k \cdot 2\pi$

$x = \frac{5}{9}\pi + k \cdot \frac{2}{3}\pi \quad \vee \quad x = \frac{1}{9}\pi + k \cdot \frac{2}{3}\pi$

Opgave 22:

a. $2\sin(2x - \frac{1}{6}\pi) = \sqrt{2}$

$\sin(2x - \frac{1}{6}\pi) = \frac{1}{2}\sqrt{2}$

$2x - \frac{1}{6}\pi = \frac{1}{4}\pi + k \cdot 2\pi \quad \vee \quad 2x - \frac{1}{6}\pi = \frac{3}{4}\pi + k \cdot 2\pi$

$2x = \frac{5}{12}\pi + k \cdot 2\pi \quad \vee \quad 2x = \frac{11}{12}\pi + k \cdot 2\pi$

$x = \frac{5}{24}\pi + k \cdot \pi \quad \vee \quad x = \frac{11}{24}\pi + k \cdot \pi$

$x = \frac{5}{24}\pi \quad \vee \quad x = \frac{11}{24}\pi \quad \vee \quad x = 1\frac{5}{24}\pi \quad \vee \quad x = 1\frac{11}{24}\pi$

b. $2\cos(3x - \frac{1}{2}\pi) = \sqrt{3}$

$\cos(3x - \frac{1}{2}\pi) = \frac{1}{2}\sqrt{3}$

$3x - \frac{1}{2}\pi = \frac{1}{6}\pi \quad \vee \quad 3x - \frac{1}{2}\pi = -\frac{1}{6}\pi$

$3x = \frac{2}{3}\pi + k \cdot 2\pi \quad \vee \quad 3x = \frac{1}{3}\pi + k \cdot 2\pi$

$x = \frac{2}{9}\pi + k \cdot \frac{2}{3}\pi \quad \vee \quad x = \frac{1}{9}\pi + k \cdot \frac{2}{3}\pi$

$x = \frac{1}{9}\pi \quad \vee \quad x = \frac{2}{9}\pi \quad \vee \quad x = \frac{7}{9}\pi \quad \vee \quad x = \frac{8}{9}\pi \quad \vee \quad x = 1\frac{4}{9}\pi \quad \vee \quad x = 1\frac{5}{9}\pi$

c. $\sin \frac{2}{3}x = -\frac{1}{2}\sqrt{2}$

$\frac{2}{3}x = -\frac{1}{4}\pi + k \cdot 2\pi \quad \vee \quad \frac{2}{3}x = 1\frac{1}{4}\pi + k \cdot 2\pi$

$x = -\frac{3}{8}\pi + k \cdot 3\pi \quad \vee \quad x = 1\frac{7}{8}\pi + k \cdot 3\pi$

$x = 1\frac{7}{8}\pi$

d. $\cos \frac{1}{2}x = -\frac{1}{2}\sqrt{3}$

$\frac{1}{2}x = \frac{5}{6}\pi + k \cdot 2\pi \quad \vee \quad \frac{1}{2}x = 1\frac{1}{6}\pi + k \cdot 2\pi$

$x = 1\frac{2}{3}\pi + k \cdot 4\pi \quad \vee \quad x = 2\frac{1}{3}\pi + k \cdot 4\pi$

$x = 1\frac{2}{3}\pi$

Opgave 23:

a. $2\sin^2 x = 1$

$\sin^2 x = \frac{1}{2}$

$\sin x = \sqrt{\frac{1}{2}} = \frac{1}{2}\sqrt{2} \quad \vee \quad \sin x = -\frac{1}{2}\sqrt{2}$

b. $\sin x = \frac{1}{2}\sqrt{2}$

$x = \frac{1}{4}\pi + k \cdot 2\pi \quad \vee \quad x = \frac{3}{4}\pi + k \cdot 2\pi$

$\sin x = -\frac{1}{2}\sqrt{2}$

$x = 1\frac{1}{4}\pi + k \cdot 2\pi \quad \vee \quad x = 1\frac{3}{4}\pi + k \cdot 2\pi$

c. de oplossingen verschillen steeds $\frac{1}{2}\pi$ van elkaar, dus $x = \frac{1}{4}\pi + k \cdot \frac{1}{2}\pi$

d. $x = \frac{1}{4}\pi$ is een oplossing en iedere andere oplossing ligt steeds $\frac{1}{2}\pi$ verder (of terug).

Opgave 24:

a. $2\cos^2(\frac{1}{2}x) = 1$

$\cos^2(\frac{1}{2}x) = \frac{1}{2}$

$\cos \frac{1}{2}x = \frac{1}{2}\sqrt{2} \quad \vee \quad \cos \frac{1}{2}x = -\frac{1}{2}\sqrt{2}$

$\frac{1}{2}x = \frac{1}{4}\pi + k \cdot 2\pi \quad \vee \quad \frac{1}{2}x = -\frac{1}{4}\pi + k \cdot 2\pi \quad \vee \quad \frac{1}{2}x = \frac{3}{4}\pi + k \cdot 2\pi \quad \vee \quad \frac{1}{2}x = -\frac{3}{4}\pi + k \cdot 2\pi$

$\frac{1}{2}x = \frac{1}{4}\pi + k \cdot \frac{1}{2}\pi$

$x = \frac{1}{2}\pi + k \cdot \pi$

b. $4\sin^2(x - \frac{1}{6}\pi) = 1$

$\sin^2(x - \frac{1}{6}\pi) = \frac{1}{4}$

$\sin(x - \frac{1}{6}\pi) = \frac{1}{2} \quad \vee \quad \sin(x - \frac{1}{6}\pi) = -\frac{1}{2}$

$x - \frac{1}{6}\pi = \frac{1}{6}\pi + k \cdot 2\pi \quad \vee \quad x - \frac{1}{6}\pi = \frac{5}{6}\pi + k \cdot 2\pi \quad \vee \quad x - \frac{1}{6}\pi = 1\frac{1}{6}\pi + k \cdot 2\pi \quad \vee$

$x - \frac{1}{6}\pi = 1\frac{5}{6}\pi + k \cdot 2\pi$

$x = \frac{1}{3}\pi + k \cdot 2\pi \quad \vee \quad x = \pi + k \cdot 2\pi \quad \vee \quad x = 1\frac{1}{3}\pi + k \cdot 2\pi \quad \vee \quad x = 2\pi + k \cdot 2\pi$

dus $x = \frac{1}{3}\pi + k \cdot \pi \quad \vee \quad x = \pi + k \cdot \pi$

c. $4\cos^2(x + \frac{1}{4}\pi) = 3$

$\cos^2(x + \frac{1}{4}\pi) = \frac{3}{4}$

$\cos(x+\frac{1}{4}\pi)=\sqrt{\frac{3}{4}}=\frac{1}{2}\sqrt{3} \quad \vee \quad \cos(x+\frac{1}{4}\pi)=-\frac{1}{2}\sqrt{3}$

$x+\frac{1}{4}\pi=\frac{1}{6}\pi+k\cdot 2\pi \quad \vee \quad x+\frac{1}{4}\pi=-\frac{1}{6}\pi+k\cdot 2\pi \quad \vee \quad x+\frac{1}{4}\pi=\frac{5}{6}\pi+k\cdot 2\pi \quad \vee$

$x+\frac{1}{4}\pi=-\frac{5}{6}\pi+k\cdot 2\pi$

$x=-\frac{1}{12}\pi+k\cdot 2\pi \quad \vee \quad x=-\frac{5}{12}\pi+k\cdot 2\pi \quad \vee \quad x=\frac{7}{12}\pi+k\cdot 2\pi \quad \vee$

$x=-1\frac{1}{12}\pi+k\cdot 2\pi$

dus $x=-\frac{1}{12}\pi+k\cdot\pi \quad \vee \quad x=\frac{7}{12}\pi+k\cdot\pi$

d. $4\sin^3 x-\sin x=0$

$\sin x(4\sin^2 x-1)=0$

$\sin x=0 \quad \vee \quad 4\sin^2 x=1$

$\sin x=0 \quad \vee \quad \sin^2 x=\frac{1}{4}$

$\sin x=0 \quad \vee \quad \sin x=\frac{1}{2} \quad \vee \quad \sin x=-\frac{1}{2}$

$x=0+k\cdot\pi \quad \vee \quad x=\frac{1}{6}\pi+k\cdot\pi \quad \vee \quad x=\frac{5}{6}\pi+k\cdot\pi$

e. $2\cos^2 x=\cos x+1$

$2\cos^2 x-\cos x-1=0$

stel $p=\cos x$

$2p^2-p-1=0$

$p=\frac{1\pm\sqrt{1+8}}{4}=\frac{1\pm 3}{4}$

$p=\frac{1+3}{4}=1 \quad \vee \quad p=\frac{1-3}{4}=-\frac{1}{2}$

$\cos x=1 \quad \vee \quad \cos x=-\frac{1}{2}$

$x=0+k\cdot 2\pi \quad \vee x=\frac{2}{3}\pi+k\cdot 2\pi \quad \vee \quad x=1\frac{1}{3}\pi+k\cdot 2\pi$

dus $x=0+k\cdot\frac{2}{3}\pi$

f. $\cos^2 x-\cos x+\frac{1}{4}=0$

$(\cos x-\frac{1}{2})^2=0$

$\cos x=\frac{1}{2}$

$x=\frac{1}{3}\pi+k\cdot 2\pi \quad \vee \quad x=-\frac{1}{3}\pi+k\cdot 2\pi$

Opgave 25:

a. $\sin\frac{1}{2}\pi x=\frac{1}{2}\sqrt{3}$

$\frac{1}{2}\pi x=\frac{1}{3}\pi+k\cdot 2\pi \quad \vee \quad \frac{1}{2}\pi x=\frac{2}{3}\pi+k\cdot 2\pi$

$x=\frac{2}{3}+k\cdot 4 \quad \vee \quad x=1\frac{1}{3}+k\cdot 4$

$x=\frac{2}{3} \quad \vee \quad x=1\frac{1}{3} \quad \vee \quad x=4\frac{2}{3} \quad \vee \quad x=5\frac{1}{3} \quad \vee \quad x=8\frac{2}{3} \quad \vee \quad x=9\frac{1}{3}$

b. $\cos\frac{1}{3}\pi x=-\frac{1}{2}\sqrt{3}$

$\frac{1}{3}\pi x=\frac{5}{6}\pi+k\cdot 2\pi \quad \vee \quad \frac{1}{3}\pi x=1\frac{1}{6}\pi+k\cdot 2\pi$

$x=2\frac{1}{2}+k\cdot 6 \quad \vee \quad x=3\frac{1}{2}+k\cdot 6$

$x=2\frac{1}{2} \quad \vee \quad x=3\frac{1}{2} \quad \vee \quad x=8\frac{1}{2} \quad \vee \quad x=9\frac{1}{2}$

c. $4\sin^2(\frac{1}{5}\pi x)=1$

$\sin^2(\frac{1}{5}\pi x)=\frac{1}{4}$

$\sin(\frac{1}{5}\pi x)=\frac{1}{2} \quad \vee \quad \sin(\frac{1}{5}\pi x)=-\frac{1}{2}$

$\frac{1}{5}\pi x = \frac{1}{6}\pi + k \cdot \pi \quad \vee \quad \frac{1}{5}\pi x = \frac{5}{6}\pi + k \cdot \pi$

$x = \frac{5}{6} + k \cdot 5 \quad \vee \quad x = 4\frac{1}{6} + k \cdot 5$

$x = \frac{5}{6} \quad \vee \quad x = 4\frac{1}{6} \quad \vee \quad x = 5\frac{5}{6} \quad \vee \quad x = 9\frac{1}{6}$

d. $2\cos^2(0{,}1\pi x) + \cos(0{,}1\pi x) = 1$

$2\cos^2(0{,}1\pi x) + \cos(0{,}1\pi x) - 1 = 0$

stel $p = \cos(0{,}1\pi x)$

$2p^2 + p - 1 = 0$

$p = \frac{-1 \pm \sqrt{1+8}}{4} = \frac{-1 \pm 3}{4}$

$p = \frac{-1+3}{4} = \frac{1}{2} \quad \vee \quad p = \frac{-1-3}{4} = -1$

$\cos(0{,}1\pi x) = \frac{1}{2} \quad \vee \quad \cos(0{,}1\pi x) = -1$

$0{,}1\pi x = \frac{1}{3}\pi + k \cdot 2\pi \quad \vee \quad 0{,}1\pi x = -\frac{1}{3}\pi + k \cdot 2\pi \quad \vee \quad x = \pi + k \cdot 2\pi$

$x = 3\frac{1}{3} + k \cdot 20 \quad \vee \quad x = -3\frac{1}{3} + k \cdot 20 \quad \vee \quad x = 10 + k \cdot 20$

$x = 3\frac{1}{3} \quad \vee \quad x = 10$

Opgave 26:

a. $x = 0{,}775 + k \cdot 2\pi \quad \vee \quad x = \pi - 0{,}775 = 2{,}366 + k \cdot 2\pi$

b. $\cos x = 0{,}8$

$x = 0{,}644 + k \cdot 2\pi \quad \vee \quad x = -0{,}644 + k \cdot 2\pi$

Opgave 27:

a. $\sin x = -0{,}85$

$x = -1{,}016 + k \cdot 2\pi \quad \vee \quad x = \pi - -1{,}016 = 4{,}158 + k \cdot 2\pi$

b. $\cos\frac{1}{2}x = 0{,}25$

$\frac{1}{2}x = 1{,}318 + k \cdot 2\pi \quad \vee \quad \frac{1}{2}x = -1{,}318 + k \cdot 2\pi$

$x = 2{,}636 + k \cdot 4\pi \quad \vee \quad x = -2{,}363 + k \cdot 2\pi$

c. $\sin(x+2) = 0{,}9$

$x + 2 = 1{,}120 + k \cdot 2\pi \quad \vee \quad x + 2 = \pi - 1{,}120 = 2{,}022 + k \cdot 2\pi$

$x = -0{,}880 + k \cdot 2\pi \quad \vee \quad x = 0{,}022 + k \cdot 2\pi$

d. $\cos(2x+1) = -0{,}4$

$2x + 1 = 1{,}982 + k \cdot 2\pi \quad \vee \quad 2x + 1 = -1{,}982 + k \cdot 2\pi$

$2x = 0{,}982 + k \cdot 2\pi \quad \vee \quad 2x = -2{,}982 + k \cdot 2\pi$

$x = 0{,}491 + k \cdot \pi \quad \vee \quad x = -1{,}491 + k \cdot \pi$

Opgave 28:

a. $2\sin 1{,}75x = 1{,}4$

$\sin 1{,}75x = 0{,}7$

$1{,}75x = 0{,}775 + k \cdot 2\pi \quad \vee \quad 1{,}75x = \pi - 0{,}775 = 2{,}366 + k \cdot 2\pi$

$x = 0{,}443 + k \cdot 3{,}59 \quad \vee \quad x = 1{,}352 + k \cdot 3{,}59$

$x = 0{,}443 \quad \vee \quad x = 1{,}352 \quad \vee \quad x = 4{,}033 \quad \vee \quad x = 4{,}943$

b. $\cos^2 0{,}95x = 0{,}86$

$\cos 0{,}95x = 0{,}927 \quad \vee \quad \cos 0{,}95x = -0{,}927$

$0{,}95x = 0{,}383 + k \cdot 2\pi \quad \vee \quad 0{,}95x = -0{,}383 + k \cdot 2\pi \quad \vee \quad 0{,}95x = 2{,}757 + k \cdot 2\pi \quad \vee$
$0{,}95x = -2{,}757 + k \cdot 2\pi$
$x = 0{,}404 + k \cdot 6{,}614 \quad \vee \quad x = -0{,}404 + k \cdot 6{,}614 \quad \vee \quad x = 2{,}902 + k \cdot 6{,}614 \quad \vee$
$x = -2{,}902 + k \cdot 6{,}614$
$x = 0{,}404 \quad \vee \quad x = 2{,}902 \quad \vee \quad x = 3{,}712 \quad \vee \quad x = 6{,}210$

Opgave 29:

a. $\sin 3x = \sin \frac{1}{6}\pi$
$3x = \frac{1}{6}\pi + k \cdot 2\pi \quad \vee \quad 3x = \frac{5}{6}\pi + k \cdot 2\pi$
$x = \frac{1}{18}\pi + k \cdot \frac{2}{3}\pi \quad \vee \quad x = \frac{5}{18}\pi + k \cdot \frac{2}{3}\pi$

b. $\cos 3x = \cos \frac{1}{6}\pi$
$3x = \frac{1}{6}\pi + k \cdot 2\pi \quad \vee \quad 3x = -\frac{1}{6}\pi + k \cdot 2\pi$
$x = \frac{1}{18}\pi + k \cdot \frac{2}{3}\pi \quad \vee \quad x = -\frac{1}{18}\pi + k \cdot \frac{2}{3}\pi$

Opgave 30:

a. $\sin(x+1) = \sin(2x+3)$
$x+1 = 2x+3 + k \cdot 2\pi \quad \vee \quad x+1 = \pi - (2x+3) + k \cdot 2\pi$
$-x = 2 + k \cdot 2\pi \quad \vee \quad x+1 = \pi - 2x - 3 + k \cdot 2\pi$
$x = -2 + k \cdot 2\pi \quad \vee \quad 3x = \pi - 4 + k \cdot 2\pi$
$x = -2 + k \cdot 2\pi \quad \vee \quad x = \frac{1}{3}\pi - \frac{4}{3} + k \cdot \frac{2}{3}\pi$

b. $\cos(2x-1) = \cos(x+1)$
$2x-1 = x+1 + k \cdot 2\pi \quad \vee \quad 2x-1 = -x-1 + k \cdot 2\pi$
$x = 2 + k \cdot 2\pi \quad \vee \quad 3x = 0 + k \cdot 3\pi$
$x = 2 + k \cdot 2\pi \quad \vee \quad x = 0 + k \cdot \frac{2}{3}\pi$

c. $\sin(2x - \frac{1}{2}\pi) = \sin(x + \frac{1}{3}\pi)$
$2x - \frac{1}{2}\pi = x + \frac{1}{3}\pi + k \cdot 2\pi \quad \vee \quad 2x - \frac{1}{2}\pi = \pi - (x + \frac{1}{3}\pi) + k \cdot 2\pi$
$x = \frac{5}{6}\pi + k \cdot 2\pi \quad \vee \quad 2x - \frac{1}{2}\pi = \pi - x - \frac{1}{3}\pi + k \cdot 2\pi$
$x = \frac{5}{6}\pi + k \cdot 2\pi \quad \vee \quad 3x = 1\frac{1}{6}\pi + k \cdot 2\pi$
$x = \frac{5}{6}\pi + k \cdot 2\pi \quad \vee \quad x = \frac{7}{18}\pi + k \cdot \frac{2}{3}\pi$

d. $\cos(x - \frac{1}{3}\pi) = \cos 2x$
$x - \frac{1}{3}\pi = 2x + k \cdot 2\pi \quad \vee \quad x - \frac{1}{3}\pi = -2x + k \cdot 2\pi$
$-x = \frac{1}{3}\pi + k \cdot 2\pi \quad \vee \quad 3x = \frac{1}{3}\pi + k \cdot 2\pi$
$x = -\frac{1}{3}\pi + k \cdot 2\pi \quad \vee \quad x = \frac{1}{9}\pi + k \cdot \frac{2}{3}\pi$

e. $\sin 2\pi x = \sin \pi (x-1)$
$2\pi x = \pi x - \pi + k \cdot 2\pi \quad \vee \quad 2\pi x = \pi - (\pi x - \pi) + k \cdot 2\pi$
$\pi x = -\pi + k \cdot 2\pi \quad \vee \quad 2\pi x = \pi - \pi x + \pi + k \cdot 2\pi$
$x = -1 + k \cdot 2 \quad \vee \quad 3\pi x = 0 + k \cdot 2\pi$
$x = -1 + k \cdot 2 \quad \vee \quad x = 0 + k \cdot \frac{2}{3}$

f. $\cos \frac{1}{2}\pi x = \cos \pi (x-2)$
$\frac{1}{2}\pi x = \pi x - 2\pi + k \cdot 2\pi \quad \vee \quad \frac{1}{2}\pi x = -\pi x + 2\pi + k \cdot 2\pi$
$-\frac{1}{2}\pi x = 0 + k \cdot 2\pi \quad \vee \quad 1\frac{1}{2}\pi x = 0 + k \cdot 2\pi$

$x = 0 + k \cdot 4 \quad \vee \quad x = 0 + k \cdot \frac{4}{3}$

dus $x = 0 + k \cdot \frac{4}{3}$

Opgave 31:

a. $\sin(2x - \frac{1}{3}\pi) = \sin(x + \frac{1}{4}\pi)$

$2x - \frac{1}{3}\pi = x + \frac{1}{4}\pi + k \cdot 2\pi \quad \vee \quad 2x - \frac{1}{3}\pi = \pi - (x + \frac{1}{4}\pi) + k \cdot 2\pi$

$x = \frac{7}{12}\pi + k \cdot 2\pi \quad \vee \quad 2x - \frac{1}{3}\pi = \pi - x - \frac{1}{4}\pi + k \cdot 2\pi$

$x = \frac{7}{12}\pi + k \cdot 2\pi \quad \vee \quad 3x = 1\frac{1}{12}\pi + k \cdot 2\pi$

$x = \frac{7}{12}\pi + k \cdot 2\pi \quad \vee \quad x = \frac{13}{36}\pi + k \cdot \frac{2}{3}\pi$

$x = \frac{13}{36}\pi \quad \vee \quad x = \frac{7}{12}\pi \quad \vee \quad x = 1\frac{1}{36}\pi \quad \vee \quad x = 1\frac{25}{36}\pi$

b. $\cos(3x + \frac{1}{2}\pi) = \cos(2x - \frac{1}{4}\pi)$

$3x + \frac{1}{2}\pi = 2x - \frac{1}{4}\pi + k \cdot 2\pi \quad \vee \quad 3x + \frac{1}{2}\pi = -2x + \frac{1}{4}\pi + k \cdot 2\pi$

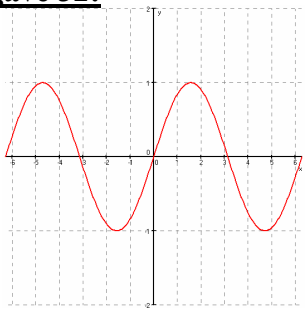
$x = -\frac{3}{4}\pi + k \cdot 2\pi \quad \vee \quad 5x = -\frac{1}{4}\pi + k \cdot 2\pi$

$x = -\frac{3}{4}\pi + k \cdot 2\pi \quad \vee \quad x = -\frac{1}{20}\pi + k \cdot \frac{2}{5}\pi$

$x = \frac{7}{20}\pi \vee \quad x = \frac{15}{20}\pi \quad \vee \quad x = 1\frac{3}{20}\pi \quad \vee \quad x = 1\frac{1}{4}\pi \quad \vee \quad x = 1\frac{11}{20}\pi \quad \vee \quad x = 1\frac{19}{20}\pi$

6.3 Transformaties bij sinusoïden

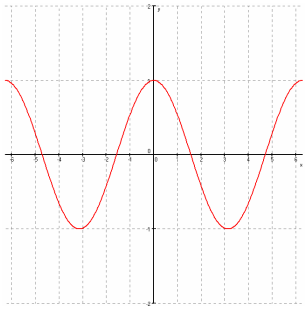
Opgave 32:

a. 

b. $(-1\frac{1}{2}\pi,1)$ $(-\frac{1}{2}\pi,-1)$ $(\frac{1}{2}\pi,1)$ $(1\frac{1}{2}\pi,-1)$

c. $(-2\pi,0)$ $(-\pi,0)$ $(0,0)$ $(\pi,0)$ $(2\pi,0)$

Opgave 33:

a. 

b. $(-2\pi,1)$ $(-\pi,-1)$ $(0,1)$ $(\pi,-1)$ $(2\pi,1)$

c. $x=-1\frac{1}{2}\pi \quad \vee \quad x=-\frac{1}{2}\pi \quad \vee \quad x=\frac{1}{2}\pi \quad \vee \quad x=1\frac{1}{2}\pi$

d. zie opgave a

Opgave 34:

a. $T(0,2)$

evenwichtsas is 2

b. $T(\frac{1}{3}\pi,0)$

$\sin(x-\frac{1}{3}\pi)=0$

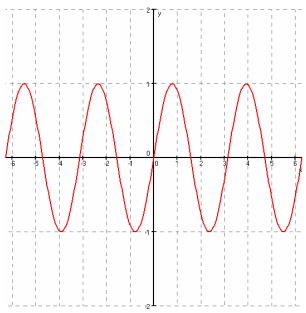
$x-\frac{1}{3}\pi=0+k\cdot\pi$

$x=\frac{1}{3}\pi+k\cdot\pi$

c. $V_{x-as,4}$

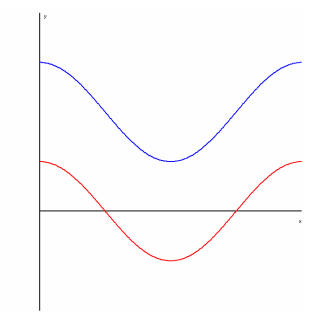
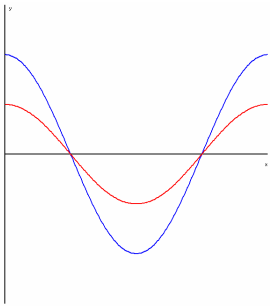
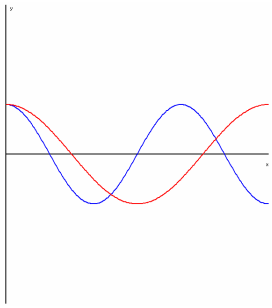
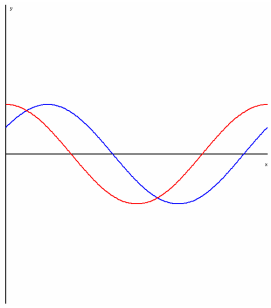
amplitude is 4

Opgave 35:

a. 

b. periode is π
c. periode is 6π

Opgave 36:

$y = a + \cos x$	$y = b\cos x$	$y = \cos cx$	$y = \cos(x - d)$
			
translatie $(0, a)$	$V_{x-as,b}$	$V_{y-as,\frac{1}{c}}$	translatie $(d,0)$
tel a op bij de functiewaarde	vermenigvuldig de functiewaarde met b	vervang x door cx	vervang x *door* $x - d$
evenwichtsas is a	amplitude is b	periode is $\frac{2\pi}{c}$	beginpunt $x = d$

Opgave 37:

a. $V_{x-as,2}$ daarna $T(-3,0)$
b. $V_{x-as,\frac{1}{3}}$ daarna $T(0,\frac{1}{5})$
c. $h(x) = \cos 3(x - 4)$
   $V_{y-as,\frac{1}{3}}$ daarna $T(4,0)$
d. $V_{x-as,1\frac{1}{2}}$ daarna $V_{y-as,4}$

Opgave 38:

a. $V_{x-as,1,2}$ daarna $T(\frac{1}{6}\pi,5)$
b. $V_{y-as,5}$ daarna $T(-\frac{1}{3}\pi;0,4)$
c. $h(x) = 0,29\cos 3(x + 1,4)$
   $V_{x-as,0,29}$ daarna $V_{y-as,\frac{1}{3}}$ daarna $T(-1,4;0)$
d. $V_{x-as,2}$ daarna $V_{y-as,\frac{1}{3}}$ daarna $T(\frac{1}{2}\pi;-0,8)$

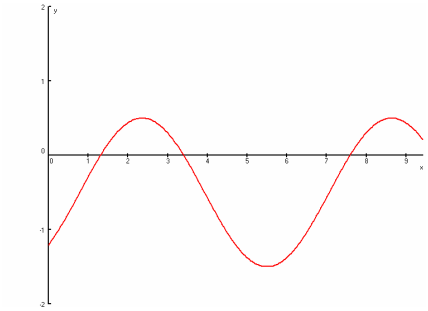
Opgave 39:

$y = \sin x \xrightarrow{V_{x-as,3}} y = \sin\frac{1}{3}x \xrightarrow{T(4;-1,5)} y = -1,5 + \sin\frac{1}{3}(x - 4)$

Opgave 40:

a. $y = \cos x \xrightarrow{T(\frac{1}{4}\pi,4)} y = 4 + \cos(x - \frac{1}{4}\pi) \xrightarrow{V_{x-as,3}} y = 12 + 3\cos(x - \frac{1}{4}\pi)$
b. $y = \cos x \xrightarrow{V_{x-as,3)}} y = 3\cos x \xrightarrow{T(\frac{1}{4}\pi;4)} y = 4 + 3\cos(x - \frac{1}{4}\pi)$

Opgave 41:

a. 

b. $(\frac{1}{4}\pi,-\frac{1}{2})$ $(1\frac{1}{4}\pi,-\frac{1}{2})$ $(2\frac{1}{4}\pi,-\frac{1}{2})$

c. $(\frac{3}{4}\pi,\frac{1}{2})$ $(1\frac{3}{4}\pi,\frac{1}{2})$ $(2\frac{3}{4}\pi,\frac{1}{2})$

d. $-\frac{1}{2}+\sin(x-\frac{1}{4}\pi)=0$

$\sin(x-\frac{1}{4}\pi)=\frac{1}{2}$

$x-\frac{1}{4}\pi=\frac{1}{6}\pi+k\cdot 2\pi \quad \vee \quad x-\frac{1}{4}\pi=\frac{5}{6}\pi+k\cdot 2\pi$

$x=\frac{5}{12}\pi+k\cdot 2\pi \quad \vee \quad x=1\frac{1}{12}\pi+k\cdot 2\pi$

$x_A=\frac{5}{12}\pi \quad x_B=1\frac{1}{12}\pi$

$AB=\frac{8}{12}\pi=\frac{2}{3}\pi$

e. $-\frac{1}{2}+\sin(x-\frac{1}{4}\pi)=-1$

$\sin(x-\frac{1}{4}\pi)=-\frac{1}{2}$

$x-\frac{1}{4}\pi=-\frac{1}{6}\pi+k\cdot 2\pi \quad \vee \quad x-\frac{1}{4}\pi=1\frac{1}{6}\pi+k\cdot 2\pi$

$x=\frac{1}{12}\pi+k\cdot 2\pi \quad \vee \quad x=1\frac{5}{12}\pi+k\cdot 2\pi$

$\frac{1}{12}\pi\le x\le 1\frac{5}{12}\pi \quad \vee \quad 2\frac{1}{12}\pi\le x\le 3\pi$

Opgave 42:

$|1+2\sin x|=2$

$1+2\sin x=2 \quad \vee \quad 1+2\sin x=-2$

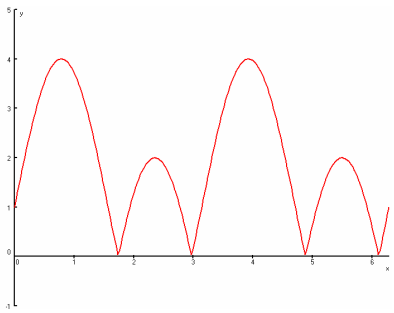
$2\sin x=1 \quad \vee \quad 2\sin x=-3$

$\sin x=\frac{1}{2} \quad \vee \quad \sin x=-1\frac{1}{2}$

$x=\frac{1}{6}\pi \quad \vee \quad x=\frac{5}{6}\pi$

$\frac{1}{6}\pi\le x\le\frac{5}{6}\pi$

Opgave 43:

a. 

b. dat zijn de toppen van $y=1+3\sin 2x$

$1+3\sin 2x=4 \quad \vee \quad 1+3\sin 2x=-2$

$3\sin 2x = 3 \quad \vee \quad 3\sin 2x = -3$

$\sin 2x = 1 \quad \vee \quad \sin 2x = -1$

$2x = \frac{1}{2}\pi + k \cdot 2\pi \quad \vee \quad 2x = 1\frac{1}{2}\pi + k \cdot 2\pi$

$x = \frac{1}{4}\pi + k \cdot \pi \quad \vee \quad x = \frac{3}{4}\pi + k \cdot \pi$

$(\frac{1}{4}\pi, 4) \quad (\frac{3}{4}\pi, 2) \quad (1\frac{1}{4}\pi, 4) \quad (1\frac{3}{4}\pi, 2)$

c. $f(\frac{1}{6}\pi) = \left|1 + 3\sin \frac{1}{3}\pi\right| = \left|1 + 3 \cdot \frac{1}{2}\sqrt{3}\right| = 1 + 1\frac{1}{2}\sqrt{3}$

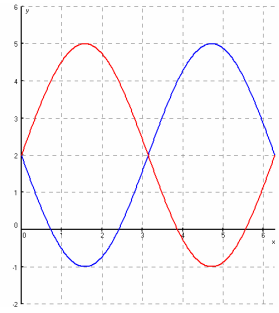
$f(\frac{1}{3}\pi) = \left|1 + 3\sin \frac{2}{3}\pi\right| = \left|1 + 3 \cdot \frac{1}{2}\sqrt{3}\right| = 1 + 1\frac{1}{2}\sqrt{3}$

$f(\frac{2}{3}\pi) = \left|1 + 3\sin \frac{4}{3}\pi\right| = \left|1 + 3 \cdot -\frac{1}{2}\sqrt{3}\right| = 1\frac{1}{2}\sqrt{3} - 1$

$f(\frac{5}{6}\pi) = \left|1 + 3\sin \frac{5}{3}\pi\right| = \left|1 + 3 \cdot -\frac{1}{2}\sqrt{3}\right| = 1\frac{1}{2}\sqrt{3} - 1$

6.4 Sinusoïden tekenen.

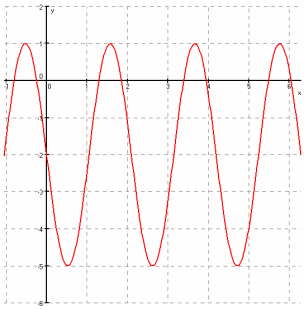
Opgave 44:

a. 

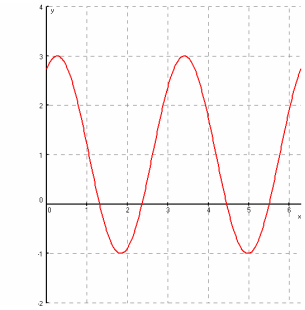
b. amplitude van f is 3
amplitude van g is 3

Opgave 45:

a. $f(x) = -2 + 3\sin 3(x + \frac{1}{3}\pi)$
evenwichtsstand: -2
amplitude: 3
periode: $\frac{2\pi}{3} = \frac{2}{3}\pi$
beginpunt: $(-\frac{1}{3}\pi, 2)$

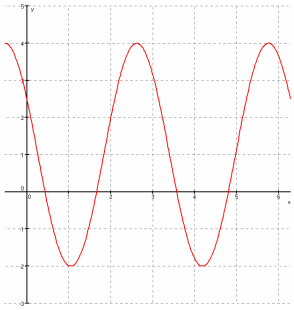


b. $g(x) = 1 - 2\sin 2(x - \frac{1}{3}\pi)$
evenwichtsstand: 1
amplitude: 2
periode: $\frac{2\pi}{2} = \pi$
beginpunt: $(\frac{1}{3}\pi, 1)$ dalend

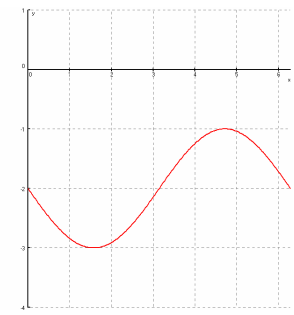


Opgave 46:

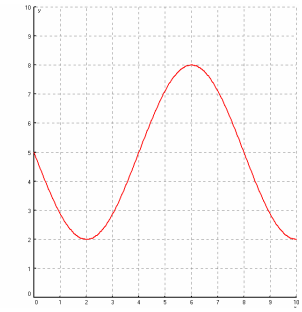
a. $f(x) = 1 + 3\cos 2(x + \frac{1}{6}\pi)$
evenwichtsstand: 1
amplitude: 3
periode: $\frac{2\pi}{2} = \pi$
beginpunt: $(-\frac{1}{6}\pi, 4)$



b. $g(x) = -2 - \cos(x - \frac{1}{2}\pi)$
evenwichtsstand: -2
amplitude: 1
periode: 2π
beginpunt: $(\frac{1}{2}\pi, -3)$ laagste punt



Opgave 47:

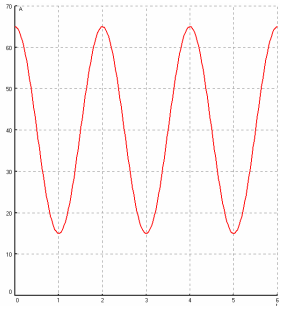


evenwichtsstand: 5
amplitude: 3
periode: $\frac{2\pi}{\frac{1}{4}\pi} = 8$
beginpunt: $(0,5)$ dalend

Opgave 48:

evenwichtsstand: 3
amplitude: 4
periode: $\frac{2\pi}{\pi} = 2$
beginpunt: $(0,-1)$ laagste punt

Opgave 49:

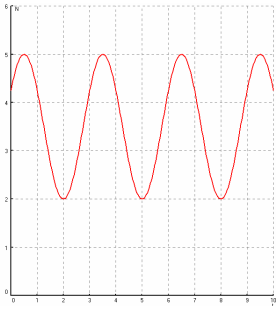


a. evenwichtsstand: 40
amplitude: 25
periode: $\frac{2\pi}{\pi} = 2$
beginpunt: $(\frac{1}{2}\pi, 40)$

b. $y_1 = 40 + 25\sin(\pi(t - 1\frac{1}{2}))$
$y_2 = 30$
intersect geeft:
$t = 0{,}63 \quad \vee \quad t = 1{,}37 \quad \vee \quad t = 2{,}63 \quad \vee \quad t = 3{,}37 \quad \vee \quad t = 4{,}63 \quad \vee \quad t = 5{,}37$
dus $0{,}63 < t < 1{,}37 \quad \vee \quad 2{,}63 < t < 3{,}37 \quad \vee \quad 4{,}63 < t < 5{,}37$

c. de grafiek snijdt de evenwichtsstand voor $t = 1\frac{1}{2}$
$\left[\frac{dy}{dx}\right]_{t=1\frac{1}{2}} = 78{,}5$

Opgave 50:



a. evenwichtsstand: $3\frac{1}{2}$
amplitude: $1\frac{1}{2}$
periode: $\frac{2\pi}{\frac{2}{3}\pi} = 3$
beginpunt: $(\frac{1}{2}, 5)$

b. $y_1 = 3{,}5 + 1{,}5\cos(\frac{2}{3}\pi(t - \frac{1}{2}))$
$y_2 = 4$
intersect geeft:
$t = 1{,}09 \quad \vee \quad t = 2{,}91 \quad \vee \quad t = 4{,}09 \quad \vee \quad t = 5{,}91 \quad \vee \quad t = 7{,}09 \quad \vee \quad t = 8{,}91$
dus: $0 \le t < 1{,}09 \quad \vee \quad 2{,}91 < t < 4{,}09 \quad \vee \quad 5{,}91 < t < 7{,}09 \quad \vee \quad 8{,}98 < t \le 10$

c. $\left[\frac{dy}{dx}\right]_{t=0} = 2{,}72$

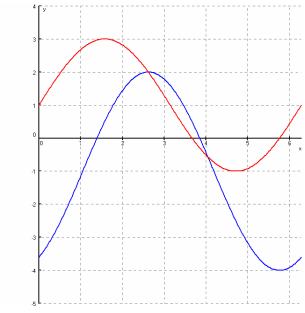
d. de grafiek snijdt de evenwichtsstand in het punt met $t = \frac{1}{2} + \frac{3}{4} \cdot 3 = 2{,}75$
$\left[\frac{dy}{dx}\right]_{t=2{,}75} = 3{,}1$

Opgave 51:
bij a hoort j
bij b hoort f
bij c hoort g
bij d hoort h

Opgave 52:
evenwichtsstand: 20
amplitude: 30
periode: 50 dus $c = \frac{2\pi}{50} = \frac{\pi}{25}$
a. $y = 20 + 30\sin\frac{\pi}{25}x$
b. $y = 20 - 30\sin(\frac{\pi}{25}(x - 25))$
c. $y = 20 + 30\cos(\frac{\pi}{25}(x - 12{,}5))$
d. $y = 20 - 30\cos(\frac{\pi}{25}(x - 37{,}5))$

Opgave 53:
evenwichtsstand: -60
amplitude: 160
periode: 6,8 dus $c = \frac{2\pi}{6{,}8} = \frac{5\pi}{17}$
a. $N = -60 + 160\sin(\frac{5\pi}{17}(t - 4))$
b. $N = -60 + 160\cos(\frac{5\pi}{17}(t - 5{,}7))$

Opgave 54:
a. f evenwichtsstand: 1
amplitude: 2
periode: 2π
beginpunt: (0,1)
g evenwichtsstand: -1
amplitude: 3
periode: 2π
beginpunt: $(\frac{1}{3}\pi, -1)$



b. $y_1 = 1 + 2\sin x$ en $y_2 = -1 + 3\sin(x - \frac{1}{3}\pi)$
intersect geeft: $x = 2{,}62 \quad \vee \quad x = 4{,}05$
$0 \le x < 2{,}62 \quad \vee \quad 4{,}05 < x \le 2\pi$
c. de evenwichtsstand van f is 1 en die van g is -1 dus is de evenwichtsstand van $f + g$ gelijk aan 0
d. teken de grafiek van $y_3 = y_1 + y_2$
evenwichtsstand: 0
amplitude: 4,36
periode: 2π
beginpunt: 0,64
$s(x) = 4{,}36\sin(x - 0{,}64)$

Opgave 55:

a. $y_1 = -3 + \cos x$ en $y_2 = \cos(x - \frac{1}{4}\pi) - 2$

neem $y_3 = y_1 + y_2$

maximum$= -2{,}202$ en minimum$= -7{,}798$

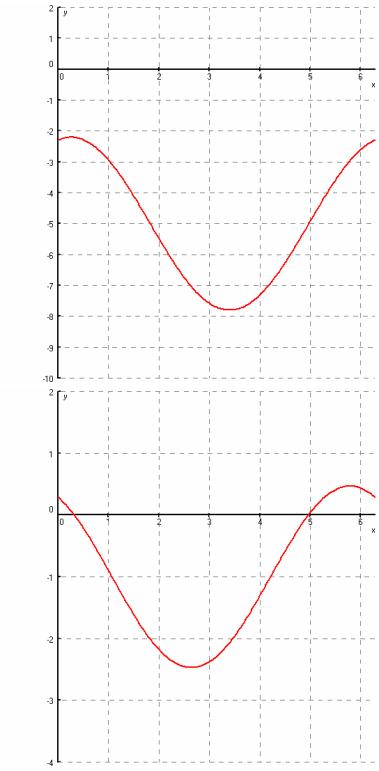
evenwichtsstand: -5

amplitude: $2{,}80$

periode: 2π

beginpunt: $(0{,}26; -2{,}20)$

$s(x) = -5 + 2{,}80\cos(x - 0{,}26)$



b. neem $y_3 = y_1 - y_2$

maximum$=0{,}474$ en minimum$= -2{,}474$

evenwichtsstand: -1

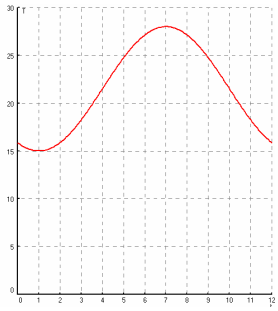
amplitude: $1{,}47$

periode: 2π

beginpunt: $(4{,}21; -1)$

$v(x) = -1 + 1{,}47\sin(x - 4{,}21)$

Opgave 56:



a. evenwichtsstand: 21,5

amplitude: 6,5

periode: $\frac{2\pi}{\frac{1}{6}\pi} = 12$

beginpunt: $(4{,}21\frac{1}{2})$

b. $y_1 = 21{,}5 + 6{,}5\sin(\frac{\pi}{6}(x-4))$ en $y_2 = 25$

intersect geeft: $t = 5{,}086 \quad \vee \quad t = 8{,}914$

$t = 5{,}086$ is 3 juni, $t = 8{,}914$ is 28 september, dus 118 dagen

of $\Delta t = 8{,}914 - 5{,}086 = 3{,}828$ maanden $= \frac{3{,}828}{12} \cdot 365 = 116$ dagen

c. $\left[\frac{dT}{dt}\right]_{t=4} = 3{,}403\,^{\circ}/_{maand} = 0{,}1\,^{\circ}/_{dag}$

d. evenwichtsstand: 17,5 dus $a = 17{,}5$

amplitude: 2,5 dus $b = 2{,}5$

periode: 12 dus $c = \frac{2\pi}{12} = \frac{\pi}{6}$

beginpunt: $t = 2 + \frac{1}{4} \cdot 12 = 5$ dus $d = 5$

$W = 17{,}5 + 2{,}5\sin(\frac{\pi}{6}(t-5))$

6.5 Diagnostische toets

Opgave 1:

a. $\sin(-270°) = \sin 90° = 1$

b. $\sin 135° = \sin 45° = \frac{1}{2}\sqrt{2}$

c. $\cos 225° = -\cos 45° = -\frac{1}{2}\sqrt{2}$

d. $\cos(-120°) = -\cos 60° = -\frac{1}{2}$

e. $\sin 240° = -\sin 60° = -\frac{1}{2}\sqrt{3}$

f. $\cos 330° = \cos 30° = \frac{1}{2}\sqrt{3}$

Opgave 2:

a. $x_P = \cos 205° = -0{,}91 \quad y_P = \sin 205° = -0{,}42$ dus $P = (-0{,}91; -0{,}42)$

b. $x_Q = \cos(-37°) = 0{,}80 \quad y_Q = \sin(-37°) = -0{,}60$ dus $Q = (0{,}80; -0{,}60)$

Opgave 3:

a. $\frac{1}{5}\pi \ rad = 36°$

b. $10\pi \ rad = 1800°$

c. $-4\pi \ rad = -720°$

d. $-4 \ rad = \frac{-4}{2\pi} \cdot 360 = -229{,}2°$

e. $\frac{2}{3}\pi \ rad = 120°$

f. $\frac{2}{3} \ rad = \frac{\frac{2}{3}}{2\pi} \cdot 360 = 38{,}2°$

Opgave 4:

a. $270° = 1\frac{1}{2}\pi \ rad$

b. $60° = \frac{1}{3}\pi \ rad$

c. $-150° = -\frac{5}{6}\pi \ rad$

d. $-135° = -\frac{3}{4}\pi \ rad$

e. $540° = 3\pi \ rad$

f. $390° = 2\frac{1}{6}\pi \ rad$

Opgave 5:

a. $26° = \frac{26}{360} \cdot 2\pi = 0{,}45 \ rad$

b. $-73° = \frac{-73}{360} \cdot 2\pi = -1{,}27 \ rad$

c. $1010° = \frac{1010}{360} \cdot 2\pi = 17{,}63 \ rad$

Opgave 6:

a. $\sin \frac{2}{7}\pi = 0{,}78$

b. $\sin \frac{2}{7} = 0{,}28$

c. $\cos 1\frac{3}{5}\pi = 0{,}31$

Opgave 7:

a. $\sin\frac{5}{6}\pi = \sin\frac{1}{6}\pi = \frac{1}{2}$

b. $\cos\frac{3}{4}\pi = -\cos\frac{1}{4}\pi = -\frac{1}{2}\sqrt{2}$

c. $\cos 1\frac{1}{3}\pi = -\cos\frac{1}{3}\pi = -\frac{1}{2}$

Opgave 8:

a. $\sin\alpha = \frac{1}{2}$

$\alpha = \frac{1}{6}\pi \quad \vee \quad \alpha = \frac{5}{6}\pi$

b. $\sin\alpha = -\frac{1}{2}\sqrt{2}$

$\alpha = 1\frac{1}{4}\pi \quad \vee \quad \alpha = 1\frac{3}{4}\pi$

c. $\cos\alpha = \frac{1}{2}\sqrt{3}$

$\alpha = \frac{1}{6}\pi \quad \vee \quad \alpha = 1\frac{5}{6}\pi$

Opgave 9:

a. $\sin(2x + \frac{1}{2}\pi) = 0$

$2x + \frac{1}{2}\pi = 0 + k \cdot \pi$

$2x = -\frac{1}{2}\pi + k \cdot \pi$

$x = -\frac{1}{4}\pi + k \cdot \frac{1}{2}\pi$

b. $\cos(2x + \frac{1}{6}\pi) = 1$

$2x + \frac{1}{6}\pi = 0 + k \cdot 2\pi$

$2x = -\frac{1}{6}\pi + k \cdot 2\pi$

$x = -\frac{1}{12}\pi + k \cdot \pi$

c. $\sin^2(\frac{1}{2}x) - \sin(\frac{1}{2}x) = 0$

$\sin(\frac{1}{2}x) \cdot (\sin(\frac{1}{2}x) - 1) = 0$

$\sin(\frac{1}{2}x) = 0 \quad \vee \quad \sin(\frac{1}{2}x) = 1$

$\frac{1}{2}x = 0 + k \cdot \pi \quad \vee \quad \frac{1}{2}x = \frac{1}{2}\pi + k \cdot 2\pi$

$x = 0 + k \cdot 2\pi \quad \vee \quad x = \pi + k \cdot 4\pi$

Opgave 10:

a. $\sin(\frac{1}{2}x + \pi) = \frac{1}{2}\sqrt{2}$

$\frac{1}{2}x + \pi = \frac{1}{4}\pi + k \cdot 2\pi \quad \vee \quad \frac{1}{2}x + \pi = \frac{3}{4}\pi + k \cdot 2\pi$

$\frac{1}{2}x = -\frac{3}{4}\pi + k \cdot 2\pi \quad \vee \quad \frac{1}{2}x = -\frac{1}{4}\pi + k \cdot 2\pi$

$x = -1\frac{1}{2}\pi + k \cdot 4\pi \quad \vee \quad x = -\frac{1}{2}\pi + k \cdot 4\pi$

b. $\cos(-\frac{1}{3}x + \frac{1}{2}\pi) = -\frac{1}{2}$

$-\frac{1}{3}x + \frac{1}{2}\pi = \frac{2}{3}\pi + k \cdot 2\pi \quad \vee \quad -\frac{1}{3}x + \frac{1}{2}\pi = -\frac{2}{3}\pi + k \cdot 2\pi$

$-\frac{1}{3}x = \frac{1}{6}\pi + k \cdot 2\pi \quad \vee \quad -\frac{1}{3}x = -1\frac{1}{6}\pi + k \cdot 2\pi$

$x = -\frac{1}{2}\pi + k \cdot 6\pi \quad \vee \quad x = -3\frac{1}{2}\pi + k \cdot 6\pi$

c. $4\cos^2(\frac{1}{2}\pi x) = 3$

$\cos^2(\frac{1}{2}\pi x) = \frac{3}{4}$

$\cos(\frac{1}{2}\pi x) = \sqrt{\frac{3}{4}} = \frac{1}{2}\sqrt{3} \quad \vee \quad \cos(\frac{1}{2}\pi x) = -\frac{1}{2}\sqrt{3}$

$\frac{1}{2}\pi x = \frac{1}{6}\pi + k \cdot 2\pi \quad \vee \quad \frac{1}{2}\pi x = 1\frac{1}{6}\pi + k \cdot 2\pi \quad \vee \quad \frac{1}{2}\pi x = \frac{5}{6}\pi + k \cdot 2\pi$

$\vee \quad \frac{1}{2}\pi x = -\frac{5}{6}\pi + k \cdot 2\pi$

$x = \frac{1}{3} + k \cdot 4 \quad \vee \quad x = -\frac{1}{3} + k \cdot 4 \quad \vee \quad x = \frac{5}{3} + k \cdot 4 \quad \vee \quad x = -\frac{5}{3} + k \cdot 4$

Opgave 11:

a. $2\sin 2x = -\sqrt{3}$

$\sin 2x = -\frac{1}{2}\sqrt{3}$

$2x = 1\frac{1}{3}\pi + k \cdot 2\pi \quad \vee \quad 2x = 1\frac{2}{3}\pi + k \cdot 2\pi$

$x = \frac{2}{3}\pi + k \cdot \pi \quad \vee \quad x = \frac{5}{6}\pi + k \cdot \pi$

$x = \frac{2}{3}\pi \quad \vee \quad x = \frac{5}{6}\pi \quad \vee \quad x = 1\frac{2}{3}\pi \quad \vee \quad x = 1\frac{5}{6}\pi$

b. $2\cos(1\frac{1}{2}x - \frac{1}{6}\pi) = -\sqrt{2}$

$\cos(1\frac{1}{2}x - \frac{1}{6}\pi) = -\frac{1}{2}\sqrt{2}$

$1\frac{1}{2}x - \frac{1}{6}\pi = \frac{3}{4}\pi + k \cdot 2\pi \quad \vee \quad 1\frac{1}{2}x - \frac{1}{6}\pi = 1\frac{1}{4}\pi + k \cdot 2\pi$

$1\frac{1}{2}x = \frac{11}{12}\pi + k \cdot 2\pi \quad \vee \quad 1\frac{1}{2}x = 1\frac{5}{12}\pi + k \cdot 2\pi$

$x = \frac{11}{18}\pi + k \cdot \frac{4}{3}\pi \quad \vee \quad x = \frac{17}{18}\pi + k \cdot \frac{4}{3}\pi$

$x = \frac{11}{18}\pi \quad \vee \quad x = \frac{17}{18}\pi \quad \vee \quad x = 1\frac{17}{18}\pi$

c. $\sin^2 x - \frac{1}{2}\sin x - \frac{1}{2} = 0$

$(\sin x - 1)(\sin x + \frac{1}{2}) = 0$

$\sin x = 1 \quad \vee \quad \sin x = -\frac{1}{2}$

$x = \frac{1}{2}\pi \quad \vee \quad x = 1\frac{1}{6}\pi \quad \vee \quad x = 1\frac{5}{6}\pi$

Opgave 12:

a. $\sin(2x - 1) = \sin(x + 2)$

$2x - 1 = x + 2 + k \cdot 2\pi \quad \vee \quad 2x - 1 = \pi - (x + 2) + k \cdot 2\pi$

$x = 3 + k \cdot 2\pi \quad \vee \quad 2x - 1 = \pi - x - 2 + k \cdot 2\pi$

$\vee \quad 3x = \pi - 1 + k \cdot 2\pi$

$\vee \quad x = \frac{1}{3}\pi - \frac{1}{3} + k \cdot \frac{2}{3}\pi$

b. $\cos(x + \frac{1}{3}\pi) = \cos(2x - \frac{1}{2}\pi)$

$x + \frac{1}{3}\pi = 2x - \frac{1}{2}\pi + k \cdot 2\pi \quad \vee \quad x + \frac{1}{3}\pi = -2x + \frac{1}{2}\pi + k \cdot 2\pi$

$-x = -\frac{5}{6}\pi + k \cdot 2\pi \quad \vee \quad 3x = \frac{1}{6}\pi + k \cdot 2\pi$

$x = \frac{5}{6}\pi + k \cdot 2\pi \quad \vee \quad x = \frac{1}{18}\pi + k \cdot \frac{2}{3}\pi$

c. $\sin(\frac{1}{2}\pi x) = \sin(\pi(x + 1))$

$\frac{1}{2}\pi x = \pi(x + 1) + k \cdot 2\pi \quad \vee \quad \frac{1}{2}\pi x = \pi - \pi(x + 1) + k \cdot 2\pi$

$\frac{1}{2}\pi x = \pi x + \pi + k \cdot 2\pi \quad \vee \quad \frac{1}{2}\pi x = \pi - \pi x - \pi + k \cdot 2\pi$

$-\frac{1}{2}\pi x = \pi + k \cdot 2\pi \quad \vee \quad 1\frac{1}{2}\pi x = 0 + k \cdot 2\pi$

$x = -2 + k \cdot 4 \quad \vee \quad x = 0 + k \cdot \frac{4}{3}$

Opgave 13:

a. $f(x) = 2\sin(3x - \frac{1}{2}\pi) = 2\sin 3(x - \frac{1}{6}\pi)$

1. $V_{y-as,\frac{1}{3}}$ 2. $V_{x-as,2}$ 3. $T(\frac{1}{6}\pi,0)$

b. 1. $V_{y-as,3}$ 2. $T(-2,5)$

c. $h(x) = 1 + 2\sin(3x - \frac{1}{4}\pi) = 1 + 2\sin 3(x - \frac{1}{12}\pi)$

1. $V_{y-as,\frac{1}{3}}$ 2. $V_{x-as,2}$ 3. $T(\frac{1}{12}\pi,1)$

Opgave 14:

$y = \sin x \xrightarrow{T(\frac{1}{2}\pi,3)} y = 3 + \sin(x - \frac{1}{2}\pi) \xrightarrow{V_{y-as,\frac{1}{5}}} y = 3 + \sin(5x - \frac{1}{2}\pi)$

Opgave 15:

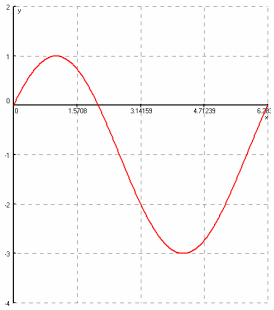
a. $f(x) = -1 + 2\cos(x - \frac{1}{3}\pi)$

evenwichts.as: -1

amplitude: 2

periode: 2π

beginpunt: $(\frac{1}{3}\pi,1)$



b. $-1 + 2\cos(x - \frac{1}{3}\pi) = -1$

$2\cos(x - \frac{1}{3}\pi) = 0$

$\cos(x - \frac{1}{3}\pi) = 0$

$x - \frac{1}{3}\pi = \frac{1}{2}\pi + k \cdot \pi$

$x = \frac{5}{6}\pi + k \cdot \pi$

$x = \frac{5}{6}\pi \quad \vee \quad x = 1\frac{5}{6}\pi$

$(\frac{5}{6}\pi,-1)$ en $(1\frac{5}{6}\pi,-1)$

c. $-1 + 2\cos(x - \frac{1}{3}\pi) = 1$

$2\cos(x - \frac{1}{3}\pi) = 2$

$\cos(x - \frac{1}{3}\pi) = 1$

$x - \frac{1}{3}\pi = 0$

$x = \frac{1}{3}\pi$

$(\frac{1}{3}\pi,1)$

$-1 + 2\cos(x - \frac{1}{3}\pi) = -3$

$2\cos(x - \frac{1}{3}\pi) = -2$

$\cos(x - \frac{1}{3}\pi) = -1$

$x - \frac{1}{3}\pi = \pi$

$x = 1\frac{1}{3}\pi$

$(1\frac{1}{3}\pi,-3)$

d. $-1 + 2\cos(x - \frac{1}{3}\pi) = 0$

$2\cos(x - \frac{1}{3}\pi) = 1$

$\cos(x - \frac{1}{3}\pi) = \frac{1}{2}$

$x - \frac{1}{3}\pi = \frac{1}{3}\pi + k \cdot 2\pi \quad \vee \quad x - \frac{1}{3}\pi = -\frac{1}{3}\pi + k \cdot 2\pi$

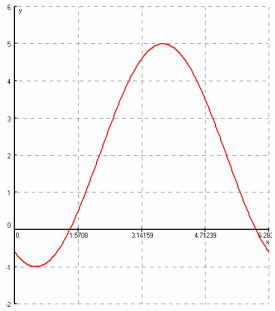
$x = \frac{2}{3}\pi + k \cdot 2\pi \quad \vee \quad x = 0 + k \cdot 2\pi$

$x = 0 \quad \vee \quad x = \frac{2}{3}\pi \quad \vee \quad x = 2\pi$ x

Opgave 16:

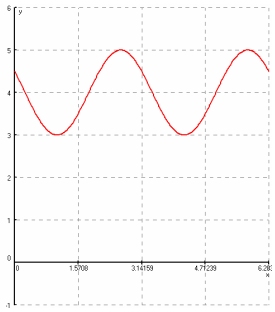
a. $f(x) = 2 - 3\sin(x + \frac{1}{3}\pi)$

evenwichts as: 2
amplitude: 3
periode: 2π
beginpunt dalend door $(-\frac{1}{3}\pi, 2)$



b. $g(x) = 4 + \cos(2x - 1\frac{2}{3}\pi) = 4 + \cos 2(x - \frac{5}{6}\pi)$

evenwichts as: 4
amplitude: 1
periode: π
beginpunt: $(\frac{5}{6}\pi, 5)$



Opgave 17:

$V = 30 + 10\sin(\frac{2}{7}\pi(t-2))$

a. evenwichts as: 30
amplitude: 10
periode: $\frac{2\pi}{\frac{2}{7}\pi} = 7$

beginpunt: $(2, 30)$

b. $y_1 = 30 + 10\sin(\frac{2}{7}\pi(x-2))$ en $y_2 = 25$

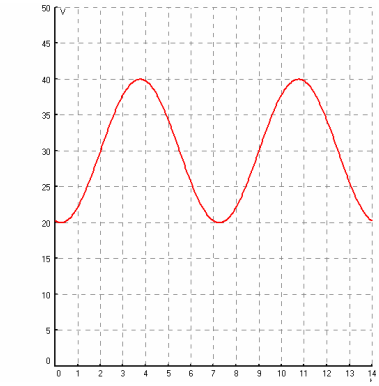
intersect geeft:

$x = 1{,}42 \quad \vee \quad x = 6{,}08 \quad \vee \quad x = 8{,}42 \quad \vee \quad x = 13{,}08$

$1{,}42 < t < 6{,}08 \quad \vee \quad 8{,}42 < t < 13{,}08$

c. dat is in het beginpunt, dus op $t = 2$

$\left[\frac{dV}{dt}\right]_{t=2} = 8{,}98$



Opgave 18:

a. minimum $= -35$
maximum $= 65$
evenwichts as: 15
amplitude: 50
periode: 30 dus $c = \frac{2\pi}{30} = \frac{1}{15}\pi$
beginpunt: $t = 25$ dalend door evenwichts-as
$N = 15 - 50\sin\frac{1}{15}\pi(t - 25)$

b. $N = 15 + 50\cos\frac{1}{15}\pi(t - 2{,}5)$

Opgave 19:

minimum $= -1{,}65$
maximum $= 3{,}65$
evenwichts as: 1
amplitude: 2,65

periode: π dus $c = \frac{2\pi}{\pi} = 2$
beginpunt: $x = 2{,}19$
$h(x) = 1 + 2{,}65 \sin 2(x - 2{,}19)$

Gemengde opgaven H6: Goniometrische formules

Opgave 13:

$x_A = \cos 40° = 0{,}766 \quad y_A = \sin 40° = 0{,}643$ dus $A = (0{,}766; 0{,}643)$

$x_B = \cos 160° = -0{,}940 \quad y_B = \sin 160° = 0{,}342$ dus $B = (-0{,}940; 0{,}342)$

$x_C = \cos 280° = 0{,}174 \quad y_C = \sin 280° = -0{,}985$ dus $C = (0{,}174; -0{,}985)$

Opgave 14:

a. $x_A = \cos \frac{2}{3}\pi = -\frac{1}{2} \quad y_A = \sin \frac{2}{3}\pi = \frac{1}{2}\sqrt{3}$ dus $A = (-\frac{1}{2}, \frac{1}{2}\sqrt{3})$

b. $x_C = \cos(-\frac{1}{6}\pi) = \frac{1}{2}\sqrt{3} \quad y_C = \sin(-\frac{1}{6}\pi) = -\frac{1}{2}$ dus $C = (\frac{1}{2}\sqrt{3}, -\frac{1}{2})$

c. $x_B = -\frac{1}{2}\sqrt{2}$ en B ligt in het derde kwadrant, dus $\beta = -\frac{3}{4}\pi$

d. cirkelboog $BC = \frac{1}{6}\pi + 1\frac{1}{4}\pi = 1\frac{5}{12}\pi$

Opgave 15:

a. bij $t = 2$ hoort $\alpha = \frac{2}{12} \cdot 2\pi = \frac{1}{3}\pi$

$x_P = \cos \frac{1}{3}\pi = \frac{1}{2} \quad y_P = \sin \frac{1}{3}\pi = \frac{1}{2}\sqrt{3}$ dus $P_{t=2} = (\frac{1}{2}, \frac{1}{2}\sqrt{3})$

bij $t = 7\frac{1}{2}$ hoort $\alpha = \frac{7\frac{1}{2}}{12} \cdot 2\pi = 1\frac{1}{4}\pi$

$x_P = \cos 1\frac{1}{4}\pi = -\cos \frac{1}{4}\pi = -\frac{1}{2}\sqrt{2} \quad y_P = \sin 1\frac{1}{4}\pi = -\sin \frac{1}{4}\pi = -\frac{1}{2}\sqrt{2}$

dus $P_{t=7\frac{1}{2}} = (-\frac{1}{2}\sqrt{2}, -\frac{1}{2}\sqrt{2})$

bij $t = 11$ hoort $\alpha = \frac{11}{12} \cdot 2\pi = 1\frac{5}{6}\pi$

$x_P = \cos 1\frac{5}{6}\pi = \cos \frac{1}{6}\pi = \frac{1}{2}\sqrt{3} \quad y_P = \sin 1\frac{5}{6}\pi = -\sin \frac{1}{6}\pi = -\frac{1}{2}$ dus $P_{t=11} = (\frac{1}{2}\sqrt{3}, -\frac{1}{2})$

b. $\cos \alpha = -\frac{1}{2}$

$\alpha = \frac{2}{3}\pi \quad \vee \quad \alpha = 1\frac{1}{3}\pi$

$t = 4 \quad \vee \quad t = 8$

Opgave 16:

a. $\cos(3x - \frac{1}{2}\pi) = \frac{1}{2}\sqrt{2}$

$3x - \frac{1}{2}\pi = \frac{1}{4}\pi + k \cdot 2\pi \quad \vee \quad 3x - \frac{1}{2}\pi = -\frac{1}{4}\pi + k \cdot 2\pi$

$3x = \frac{3}{4}\pi + k \cdot 2\pi \quad \vee \quad 3x = \frac{1}{4}\pi + k \cdot 2\pi$

$x = \frac{1}{4}\pi + k \cdot \frac{2}{3}\pi \quad \vee \quad x = \frac{1}{12}\pi + k \cdot \frac{2}{3}\pi$

b. $\sin(\frac{1}{3}x + \frac{1}{4}\pi) = -\frac{1}{2}$

$\frac{1}{3}x + \frac{1}{4}\pi = 1\frac{1}{6}\pi + k \cdot 2\pi \quad \vee \quad \frac{1}{3}x + \frac{1}{4}\pi = -\frac{1}{6}\pi + k \cdot 2\pi$

$\frac{1}{3}x = \frac{11}{12}\pi + k \cdot 2\pi \quad \vee \quad \frac{1}{3}x = -\frac{5}{12}\pi + k \cdot 2\pi$

$x = 2\frac{3}{4}\pi + k \cdot 6\pi \quad \vee \quad x = -1\frac{1}{4}\pi + k \cdot 6\pi$

c. $\sin(\frac{1}{2}x - \frac{1}{3}\pi) \cdot \cos 2x = 0$

$\sin(\frac{1}{2}x - \frac{1}{3}\pi) = 0 \quad \vee \quad \cos 2x = 0$

$\frac{1}{2}x - \frac{1}{3}\pi = 0 + k \cdot \pi \quad \vee \quad 2x = \frac{1}{2}\pi + k \cdot \pi$

$\frac{1}{2}x = \frac{1}{3}\pi + k \cdot \pi \quad \vee \quad x = \frac{1}{4}\pi + k \cdot \frac{1}{2}\pi$

$x = \frac{2}{3}\pi + k \cdot 2\pi \quad \vee \quad x = \frac{1}{4}\pi + k \cdot \frac{1}{2}\pi$

d. $4\cos^2(2\pi x - \frac{1}{2}\pi) = 3$

$\cos^2(2\pi x - \frac{1}{2}\pi) = \frac{3}{4}$

$\cos(2\pi x - \frac{1}{2}\pi) = \sqrt{\frac{3}{4}} = \frac{1}{2}\sqrt{3} \quad \vee \quad \cos(2\pi x - \frac{1}{2}\pi) = -\frac{1}{2}\sqrt{3}$

$2\pi x - \frac{1}{2}\pi = \frac{1}{6}\pi + k \cdot 2\pi \quad \vee \quad 2\pi x - \frac{1}{2}\pi = -\frac{1}{6}\pi + k \cdot 2\pi$

$\vee \quad 2\pi x - \frac{1}{2}\pi = \frac{5}{6}\pi + k \cdot 2\pi \quad \vee \quad 2\pi x - \frac{1}{2}\pi = -\frac{5}{6}\pi + k \cdot 2\pi$

$2\pi x = \frac{2}{3}\pi + k \cdot 2\pi \quad \vee \quad 2\pi x = \frac{1}{3}\pi + k \cdot 2\pi \quad \vee \quad 2\pi x = 1\frac{1}{3}\pi + k \cdot 2\pi$

$\vee \quad 2\pi x = -\frac{1}{3}\pi + k \cdot 2\pi$

$x = \frac{1}{3} + k \cdot 1 \quad \vee \quad x = \frac{1}{6} + k \cdot 1 \quad \vee \quad x = \frac{2}{3} + k \cdot 1 \quad \vee \quad x = -\frac{1}{6} + k \cdot 1$

Opgave 17:

a. $\cos(2x - \frac{1}{2}\pi) = \cos(\pi - x)$

$2x - \frac{1}{2}\pi = \pi - x + k \cdot 2\pi \quad \vee \quad 2x - \frac{1}{2}\pi = -(\pi - x) + k \cdot 2\pi$

$3x = 1\frac{1}{2}\pi + k \cdot 2\pi \quad \vee \quad 2x - \frac{1}{2}\pi = -\pi + x + k \cdot 2\pi$

$x = \frac{1}{2}\pi + k \cdot \frac{2}{3}\pi \quad \vee \quad x = -\frac{1}{2}\pi + k \cdot 2\pi$

b. $\sin(2x + \frac{1}{3}\pi) = \sin(x - \frac{1}{2}\pi)$

$2x + \frac{1}{3}\pi = x - \frac{1}{2}\pi + k \cdot 2\pi \quad \vee \quad 2x + \frac{1}{3}\pi = \pi - (x - \frac{1}{2}\pi) + k \cdot 2\pi$

$x = -\frac{5}{6}\pi + k \cdot 2\pi \quad \vee \quad 2x + \frac{1}{3}\pi = \pi - x + \frac{1}{2}\pi + k \cdot 2\pi$

$x = -\frac{5}{6}\pi + k \cdot 2\pi \quad \vee \quad 3x = 1\frac{1}{6}\pi + k \cdot 2\pi$

$x = -\frac{5}{6}\pi + k \cdot 2\pi \quad \vee \quad x = \frac{7}{18}\pi + k \cdot \frac{2}{3}\pi$

c. $\sin(\pi x) = \sin(2\pi x)$

$\pi x = 2\pi x + k \cdot 2\pi \quad \vee \quad \pi x = \pi - 2\pi x + k \cdot 2\pi$

$-\pi x = 0 + k \cdot 2\pi \quad \vee \quad 3\pi x = \pi + k \cdot 2\pi$

$x = 0 + k \cdot 2 \quad \vee \quad x = \frac{1}{3} + k \cdot \frac{2}{3}$

d. $\cos(10\pi x) = \cos(5\pi x - 6\pi)$

$10\pi x = 5\pi x - 6\pi + k \cdot 2\pi \quad \vee \quad 10\pi x = -5\pi x + 6\pi + k \cdot 2\pi$

$5\pi x = -6\pi + k \cdot 2\pi \quad \vee \quad 15\pi x = 6\pi + k \cdot 2\pi$

$x = -1\frac{1}{5} + k \cdot \frac{2}{5} \quad \vee \quad x = \frac{2}{5} + k \cdot \frac{2}{15}$

Opgave 18:

a. $\sin(1\frac{1}{2}x - \frac{1}{6}\pi) = \frac{1}{2}\sqrt{3}$

$1\frac{1}{2}x - \frac{1}{6}\pi = \frac{1}{3}\pi + k \cdot 2\pi \quad \vee \quad 1\frac{1}{2}x - \frac{1}{6}\pi = \frac{2}{3}\pi + k \cdot 2\pi$

$1\frac{1}{2}x = \frac{1}{2}\pi + k \cdot 2\pi \quad \vee \quad 1\frac{1}{2}x = \frac{5}{6}\pi + k \cdot 2\pi$

$x = \frac{1}{3}\pi + k \cdot \frac{4}{3}\pi \quad \vee \quad x = \frac{5}{9}\pi + k \cdot \frac{4}{3}\pi$

$x = \frac{1}{3}\pi \quad \vee \quad x = \frac{5}{9}\pi \quad \vee \quad x = 1\frac{2}{3}\pi \quad \vee \quad x = 1\frac{8}{9}\pi$

b. $\cos^3(2\frac{1}{2}x) + \cos(2\frac{1}{2}x) = 0$

$\cos(2\frac{1}{2}x) \cdot (\cos^2(2\frac{1}{2}x) + 1) = 0$

$\cos(2\frac{1}{2}x) = 0 \quad \vee \quad \cos^2(2\frac{1}{2}x) = -1$

$2\frac{1}{2}x = \frac{1}{2}\pi + k \cdot \pi$

$x = \frac{1}{5}\pi + k \cdot \frac{2}{5}\pi$

$x = \frac{1}{5}\pi \quad \vee \quad x = \frac{3}{5}\pi \quad \vee \quad x = \pi \quad \vee \quad x = 1\frac{2}{5}\pi \quad \vee \quad x = 1\frac{4}{5}\pi$

c. $\sin^2(1{,}5x) = \sin(1{,}5x) + 2$

$\sin^2(1{,}5x) - \sin(1{,}5x) - 2 = 0$

$(\sin(1{,}5x) - 2)(\sin(1{,}5x) + 1) = 0$

$\sin(1{,}5x) = 2 \quad \vee \quad \sin(1{,}5x) = -1$

kan niet $\quad 1{,}5x = 1\frac{1}{2}\pi + k \cdot 2\pi$

$x = \pi + k \cdot \frac{4}{3}\pi$

$x = \pi$

d. $\cos(2x + \frac{1}{3}\pi) = \cos(3x - \frac{1}{6}\pi)$

$2x + \frac{1}{3}\pi = 3x - \frac{1}{6}\pi + k \cdot 2\pi \quad \vee \quad 2x + \frac{1}{3}\pi = -3x + \frac{1}{6}\pi + k \cdot 2\pi$

$-x = -\frac{1}{2}\pi + k \cdot 2\pi \quad \vee \quad 5x = -\frac{1}{6}\pi + k \cdot 2\pi$

$x = \frac{1}{2}\pi + k \cdot 2\pi \quad \vee \quad x = -\frac{1}{30}\pi + k \cdot \frac{2}{5}\pi$

$x = \frac{11}{30}\pi \quad \vee \quad x = \frac{1}{2}\pi \quad \vee \quad x = \frac{23}{30}\pi \quad \vee \quad x = 1\frac{1}{6}\pi \quad \vee \quad x = 1\frac{17}{30}\pi \quad \vee \quad x = 1\frac{29}{30}\pi$

Opgave 19:

a. 1. $T(0,-2)$ geeft $y = -2 + \cos x$

2. $V_{x-as,3}$ geeft $y = -6 + 3\cos x$

b. 1. $T(\frac{1}{3}\pi,0)$ geeft $y = \cos(x - \frac{1}{3}\pi)$

2. $V_{y-as,\frac{1}{2}}$ geeft $y = \cos(2x - \frac{1}{3}\pi)$

c. 1. $V_{x-as,3}$ geeft $y = 3\cos x$

2. $T(0,-2)$ geeft $y = -2 + 3\cos x$

3. $V_{y-as,\frac{1}{2}}$ geeft $y = -2 + 3\cos 2x$

d. 1. $V_{y-as,\frac{1}{2}}$ geeft $y = \cos 2x$

2. $V_{x-as,3}$ geeft $y = 3\cos 2x$

3. $T(0,-2)$ geeft $y = -2 + 3\cos 2x$

e. 1. $V_{x-as,3}$ geeft $y = 3\cos x$

2. $V_{y-as,\frac{1}{2}}$ geeft $y = 3\cos 2x$

3. $T(0,-2)$ geeft $y = -2 + 3\cos 2x$

4. $V_{y-as,\frac{1}{2}}$ geeft $y = -2 + 3\cos 4x$

f. 1. $T(\frac{1}{3}\pi,0)$ geeft $y = \cos(x - \frac{1}{3}\pi)$

2. $V_{x-as,3}$ geeft $y = 3\cos(x - \frac{1}{3}\pi)$

3. $V_{y-as,\frac{1}{2}}$ geeft $y = 3\cos(2x - \frac{1}{3}\pi)$

4. $T(0,-2)$ geeft $y = -2 + 3\cos(2x - \frac{1}{3}\pi)$

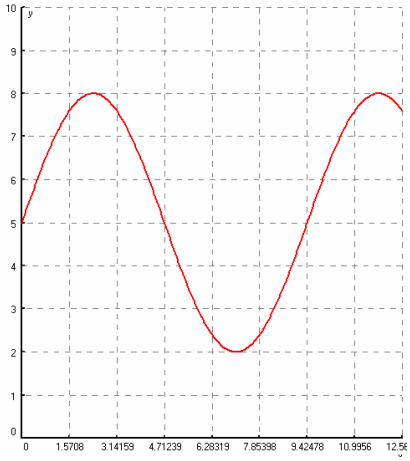
Opgave 20:

a. 1. $V_{y-as,1\frac{1}{2}}$

2. $V_{x-as,3}$

3. $T(0,5)$

b. evenwichts as: 5
amplitude: 3
periode: $\frac{2\pi}{\frac{2}{3}} = 3\pi$
beginpunt: $(0,5)$



c. $B_f = [2,8]$

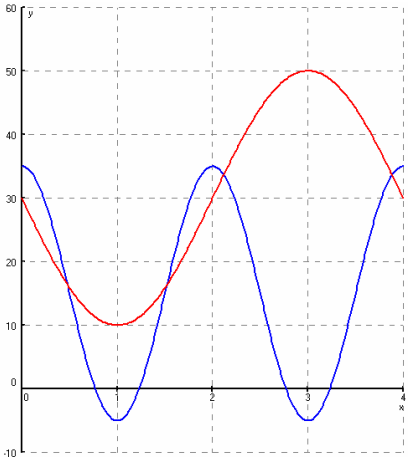
d. $y_1 = 5 + 3\sin(\frac{2}{3}x)$ en $y_2 = 3$
intersect geeft: $x = 5{,}81 \quad \vee \quad x = 8{,}33$
$0 \le x < 5{,}81 \quad \vee \quad 8{,}33 < x \le 4\pi$

e. de kleinste helling is in het punt waar de grafiek dalend door de venwichts-as gaat.
Dat is voor $x = 1\frac{1}{2}\pi$.
$\left[\frac{dy}{dx}\right]_{1\frac{1}{2}\pi} = -2$

Opgave 21:

a. f: 1. $V_{y-as,\frac{2}{\pi}}$
2. $V_{x-as,-20}$
3. $T(0,30)$

g: 1. $V_{y-as,\frac{1}{\pi}}$
2. $V_{x-as,20}$
3. $T(0,15)$



b. f: evenwichts as: 30
amplitude: 20
periode: $\frac{2\pi}{\frac{1}{2}\pi} = 4$
beginpunt: $(0,30)$ dalend

g: evenwichts as: 15
amplitude: 20
periode: $\frac{2\pi}{\pi} = 2$
beginpunt: $(0,35)$

c. $f(2{,}25) = 37{,}654$
$g(2{,}25) = 29{,}142$
$AB = 37{,}654 - 29{,}142 = 8{,}51$

d. $-5 \le p < 10$

Opgave 22:

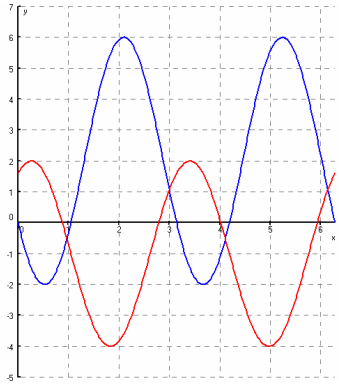
a. f: 1. $V_{y-as,\frac{1}{2}}$ 2. $V_{x-as,3}$ 3. $T(-\frac{1}{6}\pi,-1)$

$g(x) = 2 - 4\cos 2(x - \frac{1}{6}\pi)$

1. $V_{y-as,\frac{1}{2}}$ 2. $V_{x-as,-4}$ 3. $T(\frac{1}{6}\pi,2)$

b. f: evenwichts as: -1
amplitude: 3
periode: π
beginpunt: $(\frac{1}{6}\pi,-1)$

g: evenwichts as: 2
amplitude: 4
periode: π
beginpunt: $(\frac{1}{6}\pi,2)$ laagste punt



c. $y_1 = -1 + 3\sin 2(x + \frac{1}{6}\pi)$ en $y_2 = 2 - 4\cos 2(x - \frac{1}{6}\pi)$

neem $y_3 = y_2 - y_1$ de optie maximum geeft: $x = 1{,}98 \quad \vee \quad x = 5{,}12$

dus $a = 1{,}98 \quad \vee \quad a = 5{,}12$

d. neem $y_3 = y_1 + y_2$

minimum: $-1{,}05$

maximum: $3{,}05$

evenwichts as: 1

amplitude: 2,05

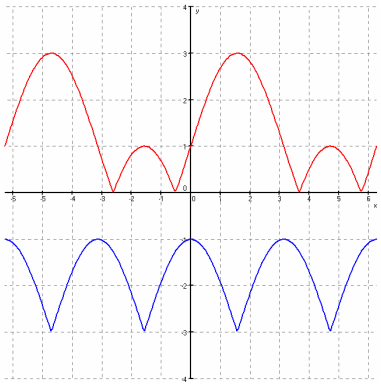
periode: π

beginpunt: $x = 1{,}72$

$h(x) = 1 + 2{,}05\sin 2(x - 1{,}72)$

Opgave 23:

a.



b. $|1 + 2\sin x| = 1$

$1 + 2\sin x = 1 \quad \vee \quad 1 + 2\sin x = -1$

$2\sin x = 0 \quad \vee \quad 2\sin x = -2$

$\sin x = 0 \quad \vee \quad \sin x = -1$

$x = 0 + k \cdot \pi \quad \vee \quad x = 1\frac{1}{2}\pi + k \cdot 2\pi$

$x = -2\pi \ \vee \ x = -\pi \ \vee \ x = -\frac{1}{2}\pi \ \vee \ x = 0 \ \vee \ x = \pi \ \vee \ x = 1\frac{1}{2}\pi \ \vee \ x = 2\pi$

c. $y_1 = abs(2\cos x) - 3$ en $y_2 = -1{,}65$ intersect geeft:

$x = -5{,}45 \vee x = -3{,}97 \vee x = -2{,}31 \vee x = -0{,}83 \vee x = 0{,}83 \vee x = 2{,}31 \vee$

$x = 3{,}97 \vee x = 5{,}45$

$-2\pi \le x < -5{,}54 \vee -3{,}97 < x < -2{,}31 \vee -0{,}83 < x < 0{,}83 \vee 2{,}31 < x < 3{,}97 \vee 5{,}45 < x \le 2\pi$

d. $f(2{,}1) = 2{,}726 \quad g(2{,}1) = -1{,}990$ dus $AB = 2{,}726 - -1{,}990 = 4{,}72$

Opgave 24:

a. $6500 - 1500 = 5000$ cm³

b. per periode: $4\frac{1}{2}$ seconde
de periode is 15 seconden
dus per minuut: $4 \cdot 4\frac{1}{2} = 18$ sec

c. G7: per periode: $4000 - 3500 = 500$ cm³ bij het inademen
de periode is 6 sec
dus per minuut: $10 \cdot 500 = 5000$ cm³
G8: $4 \cdot 5000 = 20000$ cm³
$G7 : G8 = 5000 : 20000 = 1 : 4$

d. evenwichts as: 3750
amplitude: 250
periode: 6 dus $c = \frac{2\pi}{6} = \frac{1}{3}\pi$
beginpunt: $t = 0$
$V = 3750 + 250\sin\frac{1}{3}\pi t$

e. evenwichts as: 4000
amplitude: 2500
periode: 15 dus $c = \frac{2\pi}{15} = \frac{2}{15}\pi$
beginpunt: $t = \frac{15}{4} = 3{,}75$
$V = 4000 + 2500\cos\frac{2}{15}\pi(t - 3{,}75)$

f. periode: $\frac{60}{40} = 1{,}5$ sec dus $c = \frac{2\pi}{1{,}5} = \frac{4}{3}\pi$
evenwichts as: 4200
amplitude: 2500
beginpunt: $t = 0$ laagste punt
$V = 4200 - 2500\cos\frac{4}{3}\pi t$

Opgave 25:

a. evenwichts as: $\frac{3{,}5-1{,}5}{2} = 1$
amplitude: 2,5
periode: $\frac{4}{3}\pi$ dus $c = \frac{2\pi}{\frac{4}{3}\pi} = 1\frac{1}{2}$
beginpunt: $x = \frac{1}{3}\pi$
$y = 1 + 2{,}5\sin 1\frac{1}{2}(x - \frac{1}{3}\pi)$

b. evenwichts as: 20
amplitude: 20
periode: 12 dus $c = \frac{2\pi}{12} = \frac{1}{6}\pi$
begin: $t = 1$
$N = 20 + 20\cos\frac{1}{6}\pi(t - 1)$

Opgave 26:

a. $y_1 = 3 + 3\sin(0{,}469x)$ en $y_2 = 3{,}8$ intersect geeft: $x = 0{,}576 \quad \vee \quad x = 6{,}123$
dus de breedte is: $6{,}123 - 0{,}576 = 5{,}5$ cm

b. periode: $\frac{2\pi}{0{,}469} = 13{,}4$
$\frac{67}{13{,}4} = 5$ golven
$PR = \sqrt{67^2 + 55^2} = 86{,}68$ cm
periode: $\frac{86{,}68}{5} = 17{,}34$
dus $c = \frac{2\pi}{17{,}34} = 0{,}362$
dus $y = 3 + 3\sin(0{,}362x)$